

2019

AP<sup>®</sup>

CollegeBoard

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# AP<sup>®</sup> Calculus AB

## Free-Response Questions

**2019 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB**

**SECTION II, Part A**

**Time—30 minutes**

**Number of questions—2**

**A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.**

1. Fish enter a lake at a rate modeled by the function  $E$  given by  $E(t) = 20 + 15 \sin\left(\frac{\pi t}{6}\right)$ . Fish leave the lake at a rate modeled by the function  $L$  given by  $L(t) = 4 + 2^{0.1t^2}$ . Both  $E(t)$  and  $L(t)$  are measured in fish per hour, and  $t$  is measured in hours since midnight ( $t = 0$ ).
- (a) How many fish enter the lake over the 5-hour period from midnight ( $t = 0$ ) to 5 A.M. ( $t = 5$ )? Give your answer to the nearest whole number.
- (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ( $t = 0$ ) to 5 A.M. ( $t = 5$ )?
- (c) At what time  $t$ , for  $0 \leq t \leq 8$ , is the greatest number of fish in the lake? Justify your answer.
- (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ( $t = 5$ )? Explain your reasoning.
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$t$ (hours)	0	0.3	1.7	2.8	4
$v_P(t)$ (meters per hour)	0	55	-29	55	48

2. The velocity of a particle,  $P$ , moving along the  $x$ -axis is given by the differentiable function  $v_P$ , where  $v_P(t)$  is measured in meters per hour and  $t$  is measured in hours. Selected values of  $v_P(t)$  are shown in the table above. Particle  $P$  is at the origin at time  $t = 0$ .
- (a) Justify why there must be at least one time  $t$ , for  $0.3 \leq t \leq 2.8$ , at which  $v_P'(t)$ , the acceleration of particle  $P$ , equals 0 meters per hour per hour.
- (b) Use a trapezoidal sum with the three subintervals  $[0, 0.3]$ ,  $[0.3, 1.7]$ , and  $[1.7, 2.8]$  to approximate the value of  $\int_0^{2.8} v_P(t) dt$ .
- (c) A second particle,  $Q$ , also moves along the  $x$ -axis so that its velocity for  $0 \leq t \leq 4$  is given by  $v_Q(t) = 45\sqrt{t} \cos(0.063t^2)$  meters per hour. Find the time interval during which the velocity of particle  $Q$  is at least 60 meters per hour. Find the distance traveled by particle  $Q$  during the interval when the velocity of particle  $Q$  is at least 60 meters per hour.
- (d) At time  $t = 0$ , particle  $Q$  is at position  $x = -90$ . Using the result from part (b) and the function  $v_Q$  from part (c), approximate the distance between particles  $P$  and  $Q$  at time  $t = 2.8$ .
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END OF PART A OF SECTION II

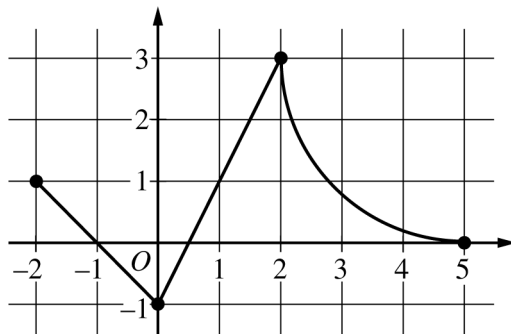
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CALCULUS AB  
SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.



Graph of  $f$

3. The continuous function  $f$  is defined on the closed interval  $-6 \leq x \leq 5$ . The figure above shows a portion of the graph of  $f$ , consisting of two line segments and a quarter of a circle centered at the point  $(5, 3)$ . It is known that the point  $(3, 3 - \sqrt{5})$  is on the graph of  $f$ .

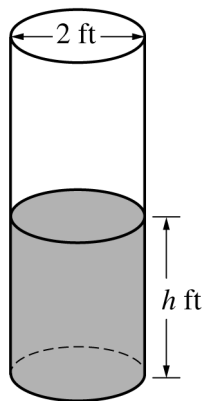
(a) If  $\int_{-6}^5 f(x) dx = 7$ , find the value of  $\int_{-6}^{-2} f(x) dx$ . Show the work that leads to your answer.

(b) Evaluate  $\int_3^5 (2f'(x) + 4) dx$ .

- (c) The function  $g$  is given by  $g(x) = \int_{-2}^x f(t) dt$ . Find the absolute maximum value of  $g$  on the interval  $-2 \leq x \leq 5$ . Justify your answer.

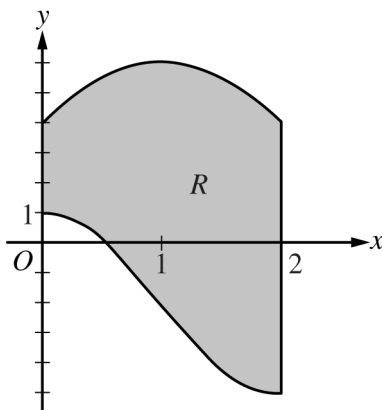
(d) Find  $\lim_{x \rightarrow 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$ .

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4. A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height  $h$  of the water in the barrel with respect to time  $t$  is modeled by  $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$ , where  $h$  is measured in feet and  $t$  is measured in seconds. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)
- Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.
  - When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.
  - At time  $t = 0$  seconds, the height of the water is 5 feet. Use separation of variables to find an expression for  $h$  in terms of  $t$ .

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5. Let  $R$  be the region enclosed by the graphs of  $g(x) = -2 + 3 \cos\left(\frac{\pi}{2}x\right)$  and  $h(x) = 6 - 2(x - 1)^2$ , the  $y$ -axis, and the vertical line  $x = 2$ , as shown in the figure above.

(a) Find the area of  $R$ .

(b) Region  $R$  is the base of a solid. For the solid, at each  $x$  the cross section perpendicular to the  $x$ -axis has

area  $A(x) = \frac{1}{x+3}$ . Find the volume of the solid.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 6$ .

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6. Functions  $f$ ,  $g$ , and  $h$  are twice-differentiable functions with  $g(2) = h(2) = 4$ . The line  $y = 4 + \frac{2}{3}(x - 2)$  is tangent to both the graph of  $g$  at  $x = 2$  and the graph of  $h$  at  $x = 2$ .

(a) Find  $h'(2)$ .

(b) Let  $a$  be the function given by  $a(x) = 3x^3h(x)$ . Write an expression for  $a'(x)$ . Find  $a'(2)$ .

(c) The function  $h$  satisfies  $h(x) = \frac{x^2 - 4}{1 - (f(x))^3}$  for  $x \neq 2$ . It is known that  $\lim_{x \rightarrow 2} h(x)$  can be evaluated using

L'Hospital's Rule. Use  $\lim_{x \rightarrow 2} h(x)$  to find  $f(2)$  and  $f'(2)$ . Show the work that leads to your answers.

(d) It is known that  $g(x) \leq h(x)$  for  $1 < x < 3$ . Let  $k$  be a function satisfying  $g(x) \leq k(x) \leq h(x)$  for  $1 < x < 3$ . Is  $k$  continuous at  $x = 2$ ? Justify your answer.

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STOP  
END OF EXAM